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## **IMPULSIVITY AND HETEROGENEITY**

***Abstract.** Preference heterogeneity occurs when discount rate varies over time, goods and decision-makers. Present paper is the first paper which formulates a reusable behavioral framework to calculate time-consistent/time-inconsistent solution under heterogeneous non-constant discount rate. Our stochastic non-constant heterogeneous quasi-hyperbolic (SNHQH) framework includes (i) SNHQH discount function, (ii) its non-standard HJB, and (iii) its behavioral equation. Our HJB has generalized all related HJBs. The SNHQH framework is prolific of usages: it can either serve a discount function or act as a tool of dimension reduction. The former usage allows us to analyze a multiple-self model, in which the financial market is driven by two Brownian motions..*

***Keywords:** time-consistent solutions, time-inconsistent preferences, sophisticated decision-maker.*

**JEL Classification: O30**

### **1.Introduction**

Overcoming the immense difficulties arising from the analytically complicated value function, this paper establishes, for the first time, a discounting-HJB-behavior framework, called SNHQH framework. As an example, the SNHQH framework is employed to study a time-inconsistent decision-maker who seeks to maximize her discounted payoff by optimally allocating her wealth.

#### **1.1. Literature**

##### **1.1.1. Literature on exponential discounting and conventional optimization**

Preference heterogeneity and time-inconsistency are by far two predominant factors effecting decision-making processes. Unfortunately, most economics and financial theories, including theories on preference heterogeneity, overlook time-inconsistency and establish themselves as exponential discounting models. These exponential discounting models are usually solved via conventional

optimization (e.g. Pontryagin Maximum Principle; standard HJB in Pham, 2009). Both exponential discounting and conventional optimization are built upon the assumptions of constant discount rates and static time preferences. These assumptions, however, have long been criticized to unrealistic (e.g. Phelps and Pollack, 1968; 2017 Nobel Prize Winner Thaler, 1981).

Because the real preferences of a decision-maker are always dynamically inconsistent (Strotz, 1955), an individual should be looked upon as a leader-follower game among successive selves. Exponential discounting is clearly counterfactual (Loewenstein and Prelec, 1992; Odonoghue and Rabin, 2015), while conventional optimization can cause distortions and misspecifications (Phelps and Pollack, 1968).

### **1.1.2. Literature on non-exponential/non-constant discounting**

Over the past quarter-century, many discount functions have been invented to translate psychological factors into computable formulas. Among them, the following discount functions translate psychological factors into computable formulas and capture time-inconsistency among selves:

(i) **SQH**. The stochastic quasi-hyperbolic (SQH) discount function in Harris and Laibson (2013) allows computing the effects of immediate gratification and is a prominent cornerstone of various papers.

(ii) **NC**. The non-constant (NC) discount function in Karp (2007) uncovers preferential factors affecting decision-making mechanism.

(iii) **SNQH**. As shown in definition 2.1, the stochastic non-constant quasi-hyperbolic (SNQH) discount function in Peng and Hager (2017) generalizes the SQH discounting and the NC discounting.

### **1.1.3. Literature on non-standard dynamic programming**

The above-mentioned discount functions have been—and can only be—solved by developing their corresponding non-standard Hamilton-Jacobi-Bellman (HJB) equations. Two methodologies are popular in deriving HJBs: (i) the continuous-time methodology in Ekeland and Pirvu (2008), Ekeland and Lazrak (2010) and Ekeland, Mbodji and Pirvu (2012) is mathematically rigorous and has distinguished resilience, depth and tightness; (ii) the discrete-time methodology in Karp (2007) is based on Euler-Maruyama (EM) discretization and opens up previously inaccessible problems.

### **1.1.4. Literature on preference heterogeneity**

Two factors contribute to preference heterogeneity. First is time heterogeneity, in which a decision-maker steeply discounts delayed rewards at the earlier phases of her life cycle and puts extra weight on a retirement pension or heritage (e.g. Myerson, 1995; Simon, 2010) as decisions are pushed into the future. Second is product heterogeneity. For example, the discount rate of one product does not apply for another product.

The preference heterogeneity is widely discussed, but rarely resolved. Under exponential discounting, the preference heterogeneity has been studied (e.g. Garleanu and Panageas, 2015). Under non-exponential and non-constant discounting, in contrast, the heterogeneity has never been captured by a discount function, let alone being controlled by a device.

**1.2. Main work**

This paper makes three contributions: (i) definition 2.1 puts forward a stochastic non-constant heterogeneous quasi-hyperbolic (SNHQH) discount function which includes all discount functions mentioned in subsection 1.1 as its special cases. (ii) theorem 2.2 uses a game-theoretic attitude to derive a reusable HJB equation, whereby a SNHQH discounter can reach time-consistency in disparate fields. (iii) sections 3--5 study a financial model in the presence of non-exponential and non-constant discounting, which otherwise is impossible when using conventional optimization (e.g. optimal control in the framework of Pontryagin and Lagrange).

**2. A computational and reusable frameworks**

This section will create the SNHQH discount function and deduce its HJB.

**2.1. Setting and problem**

To guarantee that the **SNHQH-HJB** proposed by **theorem 2.2** is applicable to both deterministic and stochastic examples, this section is constructed as follows in the stochastic setting. Throughout the whole article, it is presumed that the following setting is fulfilled. Let  $T \in (0, \infty)$  be a fixed time horizon. Let  $(\Omega, \mathcal{F}, P)$  represent a complete probability space, satisfying the usual conditions (i.e. the filtration  $\mathcal{F} = (\mathcal{F}_t)_{t \in [0, T]}$  is right-continuous and increasing;

each  $\mathcal{F}_t$  contains  $P$ -null sets in  $\mathcal{F}$ ). The norm is given by  $\|\phi\| = \left( \sum_{a=1}^{\Theta} |\phi^a|^2 \right)^{\frac{1}{2}}$ ,

for all  $\phi = (\phi^a)_{a \in \{1, 2, \dots, k\}}$ ,  $\phi \in R^{\Theta}$ ,  $\Theta \in N$ .

For  $t \in [0, \infty)$ , the cost functional is as follows

$$J(x, u, t) = E_t^x \left( \int_t^T D(t, s) L(X(s), u(s), s) ds + D(t, T) F(X(T), T) \right)$$

where (i)  $E$  is the expectation w.r.t.  $P$ .  $E_t^x = E[\cdot | X(t) = x]$  is the conditional expectation. (ii)  $L: R^d \times R^m \times [t, T] \rightarrow R$  is a utility function;  $d, m \in N = \{1, 2, \dots\}$  (iii)  $F: R^d \times [t, T] \rightarrow R$  is a terminal function. (iv)  $u: [t, T] \times \Omega \rightarrow \mathcal{U} \subseteq R^m$  is square-integrable process. The control set  $\mathcal{U} = \{u: [t, T] \times \Omega \rightarrow R^m\}$  is a compact metric space. (v)  $X: [0, T] \times \Omega \rightarrow \mathcal{X} \subseteq R^d$  is a stochastic process under the control policy  $u$ . The state set  $\mathcal{X}$  is a metric space. The endowment at time  $t$  is given by  $X(t) = x$ . For simplicity, set  $X^u := X$ . (vi)  $X(t_0) = x_0$  is the initial state at the time  $t = t_0$  and a  $R^d$ -valued random variable.  $E(\|x_0\|^l) < \infty$ , for  $l \in [1, \infty)$ . (vii)  $D(t, s)$  is a discount function,  $0 \leq t \leq s \leq T$ , with the bounds  $E(D(t, \infty)) = 0$  and  $E(D(t, t)) = 1$  for  $P$ -a.s.

For every  $(x, t) \in R^d \times [t, T]$ ,  $V$  is a value function if and only if

$$V(x, t) = \inf_{u \in \mathcal{U}} J(x, u, t)$$

where  $V: R^d \times [t, T] \rightarrow R$  is twice continuously differentiable and uniformly bounded.

The stochastic optimal control problem is

$$V(x, t) = \inf_{u \in \mathcal{U}} E_t^x \left( \int_t^T D(t, s) L(X(s), u(s), s) ds + D(t, T) F(X(T), T) \right) \quad (2.1)$$

subject to

$$\begin{cases} dX(t) = \eta(X(t), u(t), t) dt + \sigma(X(t), u(t), t) dz \\ X(t) = x \end{cases} \quad (2.2)$$

where

(i)  $z: [t, T] \times \Omega \rightarrow R^m$  is an  $m$ -dimensional Brownian motion.

(ii)  $\sigma = \left( \sigma^{a,b} \right)_{a \in \{1,2,\dots,d\}, b \in \{1,2,\dots,m\}} : R^d \times R^m \times [t, T] \rightarrow R^{d \times m}$  is a locally Lipschitz continuous function. Meanwhile, the diffusion coefficient  $\sigma$  is the infinitesimal standard deviation of the process  $X$ .

(iii)  $dX(t)$  is a  $d$ -dimensional SDE.

(iv)  $\eta = \left( \eta^a \right)_{a \in \{1,2,\dots,d\}} : R^d \times R^m \times [t, T] \rightarrow R^d$  is a locally Lipschitz continuous function whose derivative grows at most polynomially. Similarly to  $\sigma$ , the drift coefficient  $\eta$  is the infinitesimal mean of the process  $X$ .

Throughout this paper, all other processes have the dimensions and properties implied by equations (2.1)-(2.2), and all functions and stochastic processes are progressively measurable.

### 2.2. SNHQB discounting

To the best of our knowledge, equation (2.3) is the first discount function which reconciles heterogeneous, quasi-hyperbolic and non-constant preferences.

**Definition 2.1.** (SNHQB discounting). *Self  $t$  evaluates her payoff enjoyed at time  $s$  with*

$$D(t, s) = \begin{cases} D^1(t, s) = \begin{cases} \theta^1(s-t), & s \in [t, t + \zeta^1) \\ \beta^1 \theta^1(s-t), & s \in [t + \zeta^1, T) \end{cases} \\ \theta^2(T-t), & s \in [T, \infty) \end{cases} \quad (2.3)$$

where

(i)  $D^q$ : the SNQB discount function in Peng and Hager (2017) and satisfies

$$\begin{cases} D^q(t, t) = 1 \\ D^q(t, s) \in (0, 1] \end{cases} \quad \text{for } (s, t) \in [0, \infty) \quad (2.4)$$

(ii)  $\theta^q(s-t)$ : long run discount factor  $\theta^q(s-t) = e^{-\int_0^{s-t} r^q(\tau) d\tau}$

(iii)  $r(\tau)$ : a non-constant long-run discount rate. The higher the value of  $r(\tau)$ , the less a self cares about the future selves who sit in  $[t + \zeta, \infty)$ .

(iv)  $\zeta$ : an exponentially distributed random variable,  $E(\zeta) = \frac{1}{\lambda}$

(v)  $\beta$ : a present bias parameter,  $\beta \in (0,1]$

(vi)  $q$ : superscript.  $q \in \{\mathbf{1}, \mathbf{2}\}$  can be interpreted as two SNQH discounters or two products .

### 2.3. Non-standard HJB

Let  $tr$  and  $^T$  denote trace and transpose of matrices. Let  $\nabla_x V$  and  $\nabla_{xx} V$  denote the gradient of the function  $V$  and its Hessian matrix respectively. The result in the theorem 2.2 can be extended to include cases where the functions do not satisfy the regularity properties imposed in subsection 2.1 (c.f. Ngo and Taguchi, 2017).

**Theorem 2.2.** *the value function  $V(x,t) \in C^{2,1}(R^d \times [t,T])$  defined by equations (2.1)-(2.2) satisfies*

$$r(T-t)V(x,t) + K(x,t) - \frac{\partial V(x,t)}{\partial t} = \inf_{u \in \mathcal{U}} \left[ L(x,u,t) + \eta(x,u,t) \nabla_x V(x,t) - \frac{1}{2} \text{trace}(\sigma^T(x,u,t) \cdot \sigma(x,u,t) \cdot \nabla_{xx} V(x,t)) \right]$$

**Proof.** This can be proved by adapting equation (2.3) with Peng and Hager (2017) .

**Theorem 2.3. (Verification Theorem)** *Suppose  $J$  is differentiable in a  $\varepsilon$  neighborhood. For all  $\varepsilon > 0$ ,  $u^* = \varphi(s)$  is an equilibrium rule if and only if*

$$\lim_{\varepsilon \rightarrow 0} \frac{J(x,u,t) - J(\bar{x},u^\varepsilon,t)}{\varepsilon} \geq 0$$

where

$$u^\varepsilon(s) = \begin{cases} v(s) & s \in [t, t + \varepsilon] \\ \varphi(s) & s > t + \varepsilon \end{cases}$$

**Proof.** This can be proved by adapting equation (2.4) with Ekeland, Mbodji and Pirvu (2012).

Theorem 2.3 validates theorem 2.2. Providing that selves outside of the  $[t, t + \varepsilon]$  interval are kept on  $\varphi(s)$  strategy, the optimal strategy for self  $t$  in the

interval  $s \in [t, t + \varepsilon]$  is  $\varphi(s)$  strategy as well.

### 3. Application: a heterogeneous model

This section exhibits SNHQB discounting application to real life decision-making process, and disentangles heterogeneity in present bias from heterogeneity in behaviors.

#### 3.1. Heterogeneity in present bias

Our setting is a continuous-time Markovian economy with stochastic investment opportunities, allowing for a potentially incomplete market. We consider an arbitrage-free market in a complete probability space  $(\Omega, \mathcal{F}, P)$  with terminal time  $T, T > 0$ . The filtration  $\{\mathcal{F}_t\}_{t \geq 0}$  is generated by two  $m$ -dimensional Brownian motions  $Z_1$  and  $Z_2$ , satisfying the usual conditions of right-continuity and augmentation by  $P$ -null sets. A representative individual earns income from low risk assets  $Q_1$ , high risk asset  $Q_2$  and the riskless asset  $Q_3$ . The risky assets  $Q_1$  and  $Q_2$  are two  $n$  ( $\leq m$ ) dimensional stochastic processes.

$$\frac{dQ_1(t)}{Q_1(t)} = \nu dt + \sigma dZ_1 \quad \text{low-risk asset --- quantitative investment fund}$$

$$\frac{dQ_2(t)}{Q_2(t)} = \mu dt + \kappa dZ_2 \quad \text{high-risk investment---stock}$$

$$\frac{dQ_3(t)}{Q_3(t)} = \mu_0 dt \quad \text{risk-free investment---saving}$$

where  $\mu_0$  is a  $n \times m$  matrix-valued interest rate process.  $\nu$  (resp.  $\mu$ ) denotes a  $n$ -dimensional mean rate of return on quantitative investment fund (resp. stock).  $\sigma$  (resp.  $\kappa$ ) denotes a  $n \times m$  matrix-valued volatility of the quantitative investment fund (resp. stock).  $\iota$  is the covariance between  $Q_1$  and  $Q_2$ .  $p$  (resp.  $\pi$ ) is the fraction of total wealth invested in quantitative investment fund (resp. stock). Denoted by  $c$  consumption, by  $W$  wealth. For simplicity, assume  $n = m = 1$ . The profit generating process is

$$\left\{ \begin{aligned} dW(t) &= \left\{ \left[ \begin{aligned} (\nu - \mu_0) p(t) + \\ (\mu - \mu_0) \pi(t) + \mu_0 \end{aligned} \right] W(t) - c(t) \right\} dt \\ &\quad + W(t) [\sigma p(t) dZ_1(t) + \kappa \pi(t) dZ_2(t)] \\ W(t) &= w \end{aligned} \right. \quad (3.1)$$

Let  $V := V_w^{c,\pi,p}$  be a value function,  $U$  utility function,  $D$  the discount function defined in equation 2.3. An individual maximizes her utility from consumption  $U(c(t))$  and final wealth  $F(W(T), T)$ .

$$V(w, t) = E_t^w \left[ \int_t^T D(t, s) U(c(s), s) ds + D(t, T) F(W(T), T) \right] \quad (3.2)$$

### 3.2. Heterogeneity in behaviors

The decision-maker is formulated in a three-dimensional behavioral space: precommitted, naïve, sophisticated, denoted by **superscripts**  $P$ ,  $N$ ,  $S$  respectively. At time  $t_0$ , the precommitted player  $q$  and the naïve player  $q$  have the following value function (c.f. Ekeland and Pirvu, 2008; Ekeland and Lazrak, 2010; Ekeland, Mbodji and Pirvu, 2012):

$$V^{P,N}(w, t_0)$$

**Proposition 3.1.** *Under the conditions of the theorem 2.4, for  $t_0 \in [0, T]$ , the naïve and precommitted solution of equations (3.1)-(3.2) satisfy*

$$\left\{ \begin{aligned} r^{\mathbf{1}}(t_0 - t)V^{P,N}(w, t_0) &= \frac{\partial V^{P,N}(w, t_0)}{\partial t_0} + \\ \max_{c, \pi} &\left\{ \begin{aligned} &U(c(t_0)) + \left[ \begin{aligned} &(\mu - \mu_0)\pi w + (\nu - \mu_0)pw \\ &+ \mu_0 w - c(t_0) \end{aligned} \right] \frac{\partial V^{P,N}(w, t_0)}{\partial w} \\ &+ \frac{1}{2} \left[ \begin{aligned} &\sigma^2 p(t_0)^2 + 2p(t_0)\pi(t_0)l\sigma\kappa \\ &+ \kappa^2 \pi(t_0)^2 \end{aligned} \right] W^2 \frac{\partial^2 V^{P,N}(w, t_0)}{\partial w^2} \end{aligned} \right\} \quad (3.3) \\ V^{P,N}(w, T) &= F(w(T), T) \end{aligned} \right.$$

**Proposition 3.2.** *Under the conditions of the theorem 2.3, the sophisticated solution of eq.3.2 satisfies*

$$\left\{ \begin{aligned} r^{\mathbf{2}}(T - t)V^S(w, t) + K(w, t) &= \frac{\partial V^S(w, t)}{\partial t} + \\ \max_{c, \pi} &\left\{ \begin{aligned} &U(c(t)) + \left[ \begin{aligned} &(\mu - \mu_0)\pi w + \\ &(\nu - \mu_0)pw + \mu_0 w - c(t) \end{aligned} \right] \frac{\partial V^S(w, t)}{\partial w} \\ &+ \frac{1}{2} \left[ \begin{aligned} &\sigma^2 p(t)^2 + 2p(t)\pi(t)l\sigma\kappa + \kappa^2 \pi(t)^2 \end{aligned} \right] w^2 \frac{\partial^2 V^S(w, t)}{\partial w^2} \end{aligned} \right\} \quad (3.4) \\ V^S(w, T) &= F(w(T), T) \end{aligned} \right.$$

#### 4. Explicit Solutions

This section solves explicitly the optimal policies in the sophisticated, naïve and precommitted paradigms. Under logarithmic utility, the functions in equations (3.3)-(3.4) have the form of

$$\left\{ \begin{aligned} V(w, t) &= \alpha(t) \ln w + \psi(t) \\ u(c) &= \ln c \\ F(w(T), T) &= \xi \ln w \end{aligned} \right. \quad (4.1)$$

Substituting equation (4.1) into equations (3.3)-(3.4), one obtains the optimal consumption ratio

$$\frac{c(t)}{W(t)} = \frac{1}{\alpha(t)} \tag{4.2}$$

where

paradigms	$\alpha$
(i) pre-committed	$\alpha^P(t) = \frac{\theta^1(T)}{\theta^1(t)} \xi + \int_t^T \frac{\theta^1(s)}{\theta^1(t)} ds$
(ii) naive	$\alpha^N(t) = \theta^1(T-t)\xi + \int_t^T \theta^1(s-t) ds$
(iii) sophisticated	$\alpha^S(t) = \xi \theta^2(T-t) + \int_t^T \frac{\theta^2(T-t)}{\theta^2(T-s)} \omega^S(s) ds$ $\omega^S(s) = 1 - \int_s^T \theta^1(u-s) \left[ \frac{\lambda^1(1-\beta^1)e^{-\lambda^1(u-s)} + r^1(u-s) - r^2(T-s)}{r^1(u-s) - r^2(T-s)} \right] du$

### 5. Conclusion

This paper designs a pragmatic SNHQB framework to evaluate utility flows for different products/players and to calculate time-consistent solutions. The SNHQB framework incorporates many successful frameworks (e.g. SQH, NC, SNQH) as its special cases, and can be used either in the scenario of 'a single product and multiple SNQH discounter' or in the scenario of 'a single SNHQB discounter and multi-product'. Then, we develop a behavior-analyzing approach, fundamentally different from the existing ones, to discriminate heterogeneity in present bias from heterogeneity in behaviors. Furthermore, we obtain explicit solutions to a general model, and closed-form solutions for an important case analysis. Simulation confirms that the sophisticated decision-maker (who uses the non-standard HJB) enjoys higher resource consumption than the precommitted and naïve decision-makers (who use the conventional HJB).

There exists a substantial amount of future research potential in applying

the SNHQH framework to OR, economics finance, etc. Firstly, future users can model the coexistence of overconsumption and attenuated discounting of reward by building an objective function with 'a single SNHQH discounter and multi-product'. Secondly, future users can readily adapt the SNHQH framework to exponential discounting models (e.g. Garleanu and Panageas, 2015) or financial elements (e.g. hedging vehicles against the uncertainty). Thirdly, the SNHQH framework can be applied to behavioral OR setting and solved with commercial solvers.

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